

Classification of quasi-symmetric 2-(64, 24, 46) designs of Blokhuis-Haemers type

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Abstract

This paper completes the classification of quasi-symmetric 2-(64, 24, 46) designs of Blokhuis-Haemers type supported by the dual code C^\perp of the binary linear code C spanned by the lines of $AG(3, 2^2)$ initiated in [18]. It is shown that C^\perp contains exactly 30,264 nonisomorphic quasi-symmetric 2-(64, 24, 46) designs obtainable from maximal arcs in $AG(2, 2^2)$ via the Blokhuis-Haemers construction. The related strongly regular graphs are also discussed.

Keywords: quasi-symmetric design, maximal arc, linear code, automorphism group, strongly regular graph.

Mathematics Subject Classification: 05B05, 94B05.

1 Introduction

We assume familiarity with basic facts, terminology and notation from design theory, finite geometry, and coding theory [1, 2, 9, 12, 22]. For strongly regular graphs, cf. [5], [7], and for the theory of quasi-symmetric designs, one may consult the monograph [20].

In [3], Blokhuis and Haemers gave an elegant construction of a quasi-symmetric design $D(q)$ with parameters $2-(q^3, q^2(q-1)/2, q(q^3 - q^2 - 2)/4)$ and block intersection numbers $q^2(q-2)/4$ and $q^2(q-1)/4$, where q is an arbitrary power of 2. The Blokhuis-Haemers construction is a clever refinement of a method due to Shrikhande and Raghavarao [21]. Every block of $D(q)$ is the union of $q(q-1)/2$ parallel lines in the 3-dimensional affine geometry $AG(3, q)$ labeled by a block of a symmetric $2-(q^2, q(q-1)/2, q(q-2)/4)$ design invariant under the translation group of $AG(2, q)$ and defined in terms of maximal arcs in $AG(2, q)$.

In a recent paper [15], Jungnickel and Tonchev studied the properties of $D(q)$ and proved that the number of nonisomorphic quasi-symmetric designs obtainable via the Blokhuis-Haemers construction grows exponentially with linear growth of q . Following [15], we call any design obtained via the Blokhuis-Haemers construction a BH-design.

In particular, if $q = 4$, it was proved in [15, Proposition 3.7] that there are at least 28,844 nonisomorphic $2-(64, 24, 46)$ BH-designs. In this paper, we give a complete classification of all $2-(64, 24, 46)$ BH-designs up to isomorphism, and show that the total number of nonisomorphic such designs is exactly 30,264 (Theorem 2). This classification is a continuation of the work in [18]. Our approach employs a binary linear code associated with the designs in question that utilizes the following property of BH-designs: if $q > 2$, every block of $D(q)$ meets every line of $AG(3, q)$ in an even number of points [3], [15, Lemma 3.2, (d)]. This property implies the following.

Lemma 1 [18]. *If $q > 2$, every block of $D(q)$ is the support of a codeword of weight $q^2(q-1)/2$ in the dual code C^\perp of the binary code C of length q^3 spanned by the incidence vectors of the lines in $AG(3, q)$.*

If $q = 4$, the binary code C spanned by the lines of $AG(3, 4)$ is of dimension 51 (by Hamada's rank formula [10]), hence the dimension of C^\perp is 13. The weight enumerator $W(x)$ of C^\perp is

$$W(x) = 1 + 1008x^{24} + 6174x^{32} + 1008x^{40} + x^{64}.$$

The automorphism group $G = \text{Aut}(C^\perp)$ of C^\perp coincides with the collineation group $\Gamma L(3, 4)$ of $AG(3, 4)$, and is of order

$$23, 224, 320 = 2 \times 4^3(4^3 - 1)(4^3 - 4)(4^3 - 4^2) = 2^{13} \cdot 3^4 \cdot 5 \cdot 7. \quad (1)$$

We use the code C^\perp to find block-by-point incidence matrices of quasi-symmetric $2-(64, 24, 46)$ BH-designs as collections of 336 codewords of weight 24, such that every

two codewords share either 8 or 12 nonzero positions. For these computations, we used Magma [4] and Cliquer [17].

In [18], this approach was used to classify up to isomorphism all BH-designs invariant under automorphisms of C^\perp of odd prime order. It was shown that there is exactly one isomorphism class of designs admitting automorphisms of order 7, fifteen isomorphism classes of designs admitting automorphisms of order 5, and no designs with automorphisms of order 3. In addition, it was shown that there is exactly one BH-design in C^\perp with a full automorphism group being the Sylow 2-subgroup of G of order 2^{13} .

It is the goal of this paper to complete the classification of 2-(64, 24, 46) BH-designs, by finding representatives of the isomorphism classes of all remaining designs, which must have full automorphism groups of order 2^i for $i < 13$.

2 Counting 2-(64, 24, 46) BH-designs

The following lemmas were crucial in making the classification of 2-(64, 24, 46) BH-designs computationally feasible.

Lemma 2 *The total number of distinct 2-(64, 24, 16) BH-designs is 3^{21} .*

Proof. We consider two designs D_1, D_2 to be distinct if their collections of blocks are distinct, that is, there is a block B_1 of D_1 which is not a block of D_2 . The statement of the lemma follows from [15, Theorem 2.2], and can also be verified computationally as follows.

Let D' be the 2-(64, 4, 1) design of the lines in $AG(3, 4)$, and consider the natural resolution of D' into 21 parallel classes of lines, where each parallel class consists of a line through the origin and its translates. Let P be such a parallel class, consisting of lines L_1, \dots, L_{16} , and let D'' be a symmetric 2-(16, 6, 2) design such that the points of D'' are labeled with the sixteen lines from P . The Blokhuis-Haemers construction (or more generally, the Shrikhande-Raghavarao construction [21]) replaces the 16 lines of P with 16 new blocks of size 24, each being a union

$$U = L_{i_1} \cup L_{i_2} \cup \dots \cup L_{i_6}$$

of six lines of P that correspond to a block of D'' . Clearly, any two of the sixteen new blocks of size 24 share two lines from P , hence meet each other in 8 points. If D'' is appropriately chosen design obtained from maximal arcs in $AG(2, 4)$ (cf. [15]), the resulting new design with blocks of size 24 is a quasi-symmetric 2-(64, 24, 46) design [3], [15].

By Lemma 1, every block of a BH-design meets every line of $AG(3, 4)$ in an even number (0, 2 or 4) of points. It is easy to check by computer that there are exactly

48 unions, U_1, U_2, \dots, U_{48} , of six lines from P that meet every line of $AG(3, 4)$ evenly. We define a graph Γ_P with vertices U_1, U_2, \dots, U_{48} , where U_i and U_j are adjacent if they share either 8 or 12 points. A quick check shows that the maximum clique size in Γ_P is 16, and there are exactly three 16-cliques. The three 16-cliques are mutually disjoint, that is, partition the vertex set $\{U_i\}_{i=1}^{48}$. In addition, every 16-clique consists of blocks that meet each other in exactly 8 points. We call the three 16-cliques associated with a parallel class P *special* cliques. The symmetric $2-(16, 6, 2)$ design D'' associated with any special clique is invariant under the elementary abelian group E_{16} of order 16 acting transitively on the blocks of D'' and the set of lines of P . The blocks of D'' are maximal arcs in an affine plane of order 4, $AG(2, 4)$, associated with P [15]. The design D'' is isomorphic to the unique $2-(16, 6, 2)$ design admitting a 2-transitive automorphism group, and is also the unique SDP design with these parameters, in the terminology of [14, 16].

Since the collineation group of $AG(3, 4)$ is transitive on the set of 21 parallel classes of lines, the same applies to each parallel class. It follows that the collection of blocks of any BH-design is a union of 21 special cliques, one clique for each of the 21 parallel classes. This implies that the number of BH-designs is at most 3^{21} .

To prove the equality, we compute a graph Δ having 1008 vertices corresponding to the blocks of size 24 associated with the 21 parallel classes (48 blocks per parallel class). We define two blocks to be adjacent if they share 8 or 12 points. A quick computer check shows that any two blocks associated with different parallel classes are adjacent in Δ . Consequently, every collection of 21 special cliques, one for each of the 21 different parallel classes, is the set of blocks of a BH-design. \square

Note 1 The 1008 blocks of size 24 in Lemma 2 correspond to the 1008 codewords of weight 24 in the code C^\perp (cf. (1)).

Lemma 3 *Every $2-(64, 24, 46)$ BH-design is invariant under an elementary abelian group of order 64, isomorphic to the translation group T of $AG(3, 4)$.*

Proof. The translation group T stabilizes each parallel class P of $AG(3, 4)$ and the special cliques associated with P . For a geometric proof of an analogous result for arbitrary q see [15]. \square

Since the automorphism group $G \cong \Gamma L(3, 4)$ of C^\perp preserves the partition of the set of 1008 codewords into triples of special 16-cliques associated with the 21 parallel classes of lines in $AG(3, 4)$, we have the following.

Lemma 4 *The automorphism group of every BH-design is a subgroup of $G = \text{Aut}(C^\perp) \cong \Gamma L(3, 4)$.*

Lemma 4 also follows from [15, Lemma 3.4].

Suppose that the number of nonisomorphic BH-designs in C^\perp is N , and let D_1, \dots, D_N be a set of N pairwise nonisomorphic BH-designs. As a corollary of Lemmas 2, 3, and 4, we have the following equation:

$$3^{21} = \sum_{i=1}^N \frac{|G|}{|Aut(D_i)|}, \quad (2)$$

where $Aut(D_i)$ denotes the full automorphism group of D_i .

We can split the right-hand side sum in (2) into two parts:

$$\sum_{i=1}^N \frac{|G|}{|Aut(D_i)|} = \frac{|G|}{64} N_{64} + \sum_{j: 128 \mid |Aut(D_j)|} \frac{|G|}{|Aut(D_j)|}, \quad (3)$$

where N_{64} denotes the number of nonisomorphic designs with full group of order 64.

Thus, if we find the number of nonisomorphic BH-designs having automorphism group of order divisible by $2^7 = 128$, we can determine N_{64} and N from equations (2) and (3), which would complete the classification of 2-(64, 24, 16) designs.

3 Classifying BH-designs with a group of order 128

It is known that every finite group of order 2^n contains a subgroup of order 2^i for every i in the range $1 \leq i \leq n$ (cf. [19, Theorem 6.5, page 116]). By this property, finding all nonisomorphic BH-designs in C^\perp which are invariant under a subgroup of $G = Aut(C^\perp)$ of order 128, will complete the classification of BH-designs in C^\perp .

By Lemma 3, it suffices to consider only the subgroups of G which contain the translation group T of order 64. Using Magma, we found that the group G contains 962 conjugacy classes of subgroups of order 128, but only two contain representatives that contain T as a subgroup. In what follows, we will use two such subgroups of G , denoted by H_1 and H_2 .

The group H_1 is isomorphic to the group labeled by (128, 2163) in the Magma small groups library [4]. The normalizer $N_G(H_1)$ of H_1 in G has order 73728. We use the normalizer $N_G(H_1)$ for elimination of isomorphic designs.

The group H_1 partitions the 1008 codewords of C^\perp of weight 24 into 39 orbits: 15 orbits of length 16, and 24 orbits of length 32. We call an orbit *good* if any two codewords from that orbit share exactly 8 or 12 nonzero positions. It turns out that all H_1 -orbits are good, hence all orbits could be used to build a BH-design with parameters 2-(64, 24, 46). We call two orbits *compatible* if every codeword from one orbit shares exactly 8 or 12 nonzero positions with every codeword from the other orbit. We define a graph Γ with 39 vertices corresponding to the H_1 -orbits, two vertices being adjacent if and only if the corresponding orbits are compatible. Every BH-design invariant

under H_1 corresponds to a clique in Γ labeled by a set of pairwise compatible orbits containing a total of 336 codewords. Let Γ_1 (resp. Γ_2) be the subgraph of Γ having as vertices the 15 orbits of length 16 (resp. the 24 orbits of length 32). The maximum clique size in Γ_1 is 5, while the maximum clique size in Γ_2 is 8. It follows that any BH-design invariant under H_1 corresponds to the union of one 5-clique from Γ_1 and one 8-clique from Γ_2 . A further check shows that every H_1 -orbit of length 16 is compatible with all H_1 -orbits of length 32. Thus, the union of each 5-clique of Γ_1 with an 8-clique Γ_2 gives a BH-design.

The normalizer $N_G(H_1)$ acts in two orbits on the set of 15 H_1 -orbits of length 16, while it acts transitively on the set of 24 H_1 -orbits of length 32. Hence, it is sufficient to consider only designs that contain one fixed H_1 -orbit of length 32, for example, the first such orbit. For further elimination of isomorphic designs, we use the stabilizer of the first H_1 -orbit of length 32 in the group $N_G(H_1)$.

A computation based on this approach shows that there are exactly 2688 mutually nonisomorphic BH-designs that admit H_1 as an automorphism group. Information about the orders of the full automorphism groups of these designs, and the number of nonisomorphic designs with a full automorphism group of given order, is given in Table 1.

| $ \text{Aut}(\mathcal{D}) $ | # designs | $ \text{Aut}(\mathcal{D}) $ | # designs | $ \text{Aut}(\mathcal{D}) $ | # designs |
|-----------------------------|-----------|-----------------------------|-----------|-----------------------------|-----------|
| 20480 | 2 | 1280 | 1 | 512 | 64 |
| 8192 | 1 | 1024 | 8 | 256 | 210 |
| 2048 | 3 | 640 | 12 | 128 | 2387 |

Table 1: Nonisomorphic 2-(64, 24, 46) BH-designs admitting H_1 as an automorphism group

The second subgroup H_2 of order 128 which contains T , is isomorphic to the group labeled by (128, 1578) in the Magma small groups library. The normalizer $N_G(H_2)$ has order 21504. H_2 partitions the 1008 codewords of weight 24 into 35 orbits: 7 orbits of length 16, and 28 orbits of length 32. All H_2 -orbits of length 16 are good, and 21 of the 28 orbits of length 32 are good orbits. As in the case with H_1 , we define a graph Γ with 28 vertices corresponding to good H_2 -orbits, two vertices being adjacent if and only if the corresponding orbits are compatible, and search for cliques that determine 2-(64, 24, 46) BH-designs. We define graphs Γ_1 and Γ_2 as induced subgraphs of Γ with vertex sets determined by orbits of length 16 or 32, respectively. Γ_1 is a complete graph on 7 vertices, and the maximum size of a clique in Γ_2 is 7. Any H_2 -orbit of length 16 is compatible with all good H_2 -orbits of length 32, hence any 7-clique in Γ_2 together with the vertices of Γ_1 determines a BH-design with parameters 2-(64, 24, 46). There are exactly 2187 7-cliques in Γ_2 , yielding 2187 distinct 2-(64, 24, 46) designs. In

that set of 2187 BH-designs, there are 17 mutually nonisomorphic ones. Information about the orders of the full automorphism groups of these designs, and the number of nonisomorphic designs with a full automorphism group of given order, is listed in Table 2.

| $ \text{Aut}(\mathcal{D}) $ | # designs | $ \text{Aut}(\mathcal{D}) $ | # designs |
|-----------------------------|-----------|-----------------------------|-----------|
| 8192 | 1 | 512 | 3 |
| 1024 | 1 | 256 | 1 |
| 896 | 1 | 128 | 10 |

Table 2: Nonisomorphic 2-(64, 24, 46) BH-designs admitting H_2 as an automorphism group

The full automorphism groups of order 8192, 1024, 512 and 256 contain both groups H_1 and H_2 , hence among the 17 BH-designs invariant under H_2 there are 6 designs that admit H_1 as well. Thus, there are exactly 2699 nonisomorphic designs among the 2-(64, 24, 46) BH-designs summarized in Table 1 and Table 2. We note that the 2-(64, 24, 46) BH-designs found in [18] have full automorphism groups of orders $20480 = 2^{12} \cdot 5$, $8192 = 2^{13}$, $1280 = 2^8 \cdot 5$, $896 = 2^7 \cdot 7$, and $640 = 2^7 \cdot 5$. Thus, all designs from [18] admit also an automorphism group of order 128. Hence, we have the following.

Theorem 1 *There are exactly 2699 nonisomorphic 2-(64, 24, 46) BH-designs admitting an automorphism group of order 128.*

If \mathcal{D} is a 2-(64, 24, 46) BH-design that does not admit an automorphism group of order 128, then the translation group T of $AG(3, 4)$ is its full automorphism group.

Theorem 1 and the results from Section 2 imply the main result of this paper.

Theorem 2 *There are exactly 30,264 nonisomorphic BH-designs with parameters 2-(64, 24, 46).*

Proof. Using the data from Tables 1 and 2, we have

$$\sum_{j: 128 \nmid |\text{Aut}(\mathcal{D}_j)|} \frac{|\text{Aut}(C^\perp)|}{|\text{Aut}(\mathcal{D}_j)|} = 457566003. \quad (4)$$

whence, from equations (2) and (3) we have

$$3^{21} = \sum_{i=1}^N \frac{|\text{Aut}(C^\perp)|}{|\text{Aut}(\mathcal{D}_i)|} = \frac{23224320}{64} N_{64} + 457566003.$$

Hence, $N_{64} = 27565$ and $N = 30264$. \square

Information about the orders of the full automorphism groups of the 30264 nonisomorphic BH-designs, and the number of nonisomorphic designs with a group of given order, is listed in Table 3.

| $ \text{Aut}(\mathcal{D}) $ | # designs | $ \text{Aut}(\mathcal{D}) $ | # designs | $ \text{Aut}(\mathcal{D}) $ | # designs |
|-----------------------------|-----------|-----------------------------|-----------|-----------------------------|-----------|
| 20480 | 2 | 1024 | 8 | 256 | 210 |
| 8192 | 1 | 896 | 1 | 128 | 2397 |
| 2048 | 3 | 640 | 12 | 64 | 27565 |
| 1280 | 1 | 512 | 64 | | |

Table 3: Nonisomorphic BH-designs with parameters 2-(64, 24, 46)

Among the 2699 nonisomorphic BH-designs with parameters 2-(64,24,46) admitting an automorphism group of order 128, there are three designs having 2-rank equal to 12, namely the design with full automorphism group of order 8192, one of the designs with full automorphism group of order 2048, and one of the designs with full automorphism group of order 20480. All other designs have 2-rank 13. A list of the nonisomorphic 2-(64, 24, 46) BH-designs admitting an automorphism group of order 128 is available at

www.math.uniri.hr/~sanjar/structures/.

4 Strongly regular graphs with parameters (336,80,28,16)

A *strongly regular graph* with parameters (n, k, λ, μ) is a graph with n vertices which is regular of degree k , every two adjacent vertices have λ common neighbors, and every two nonadjacent vertices have μ common neighbors. The block graph of a quasi-symmetric 2-(64, 24, 46) design with block intersection numbers 8 and 12, where two blocks are adjacent if they share 12 points, is a strongly regular graph with parameters (336, 80, 28, 16). These are also the parameters of the block graph of any Steiner 2-(64, 4, 1) design, where two blocks are adjacent if they share a point. The graphs obtained from a quasi-symmetric 2-(64, 24, 46) BH-design and a resolvable Steiner 2-(64, 4, 1) design (for example, the 2-(64, 4, 1) design of the lines of $AG(3, 4)$) share the property that their sets of vertices can be partitioned into 21 cocliques of size 16. Strongly regular graphs whose point set can be partitioned into cliques (or cocliques), are studied in [11]. A (336, 80, 28, 16) strongly regular graph which is not the block

graph of a Steiner or a quasi-symmetric design is discussed in [6] and [13] (see also the on-line table of strongly regular graphs maintained by Andries Brouwer [5]).

The block graphs of the 2699 nonisomorphic 2-(64, 24, 46) BH-designs admitting an automorphism of order 128, split into 2371 isomorphism classes of strongly regular graphs with parameters (336, 80, 28, 16).

Information about orders of the full automorphism groups of these strongly regular graphs is given in Table 4.

| $ \text{Aut}(\Gamma) $ | # SRGs | $ \text{Aut}(\Gamma) $ | # SRGs |
|------------------------|--------|------------------------|--------|
| 245760 | 1 | 1280 | 4 |
| 61440 | 1 | 1024 | 12 |
| 24576 | 1 | 896 | 1 |
| 8192 | 1 | 640 | 7 |
| 6144 | 1 | 512 | 56 |
| 4096 | 2 | 384 | 4 |
| 2048 | 6 | 256 | 220 |
| 1536 | 3 | 128 | 2051 |

Table 4: Nonisomorphic block graphs of 2-(64, 24, 46) BH-designs admitting an automorphism group of order 128

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